Beliefs and attitudes in mathematics teaching and learning

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Introduction

Beliefs play great role in mathematics learning and teaching. The learning outcomes of students are strongly related to their beliefs and attitudes about mathematics (Furinghetti & Pehkonen, 2000). Thus assessing or evaluating of students’ mathematical knowledge must be made in awareness of their beliefs. During the last twenty years the research area about beliefs and attitudes has grown considerably, and many different countries have been included in the research - the work of Erkki Pehkonen who has made several investigations in Finland; Peter Kloosterman from United States of America, Günter Törner from Germany, Gilah C. Leder from Australia, and many others. They have been looking for answers to several different questions. For example, Kloosterman asked (2002): “What do students think mathematics is and how does one learn mathematics?”; Pehkonen & Törner (2004) asked: “How well does information from different methodological sources and using different methodological tools to investigate teachers’ beliefs of mathematics fit together?” and “Which method is best suited to which aspect?”.

Before introducing our study we would like to emphasise the work of Alba G. Thompson who has been investigating teachers’ beliefs extensively. She has looked into teachers’ conceptions of mathematics and their relationship to instructional practice. The issue of changing teachers’ beliefs, comments on methodology, and descriptions of theoretical frameworks, are included in her work (Thompson, 1992).

In our project the issue about students’ beliefs are of interest because we want to find answers to questions about what Norwegian and Estonian students think about mathematics as a school subject, about the way one learns mathematics, and what is important in the study of mathematics. In this paper we report from a small pilot study carried out in Norway and an earlier study in Norway, which it is based upon.

Since 1995 the project KIM has collected national data on students’ understanding of key concepts in the national mathematics curriculum. Students’ performances related to one particular area of mathematics, named Measurements and units, were linked to their attitudes. 105 grade six classes (2106 students) and 90 grade nine classes (2150 students) took part. Amongst those students approximately 900 were selected according to their birthdays. The study is based on data from 891 grade 6 students and 893 students in grade 9. The same schools were asked to participate in the attitude study later in that school year, which made it possible to compare the mathematics test to the questions about their thoughts of mathematics and the teaching and learning mathematics. The conclusions of this project were following: those pupils who state a positive interest towards mathematics, on average, performed better on the mathematics test than their fellow students. There was a strong significant connection between the performance of the test and the self-confidence in both grades (Streitlien, Wiik & Brekke, 2001).
Definition of beliefs

Leder and Forgasz claimed that

In everyday language, the term “belief” is often used loosely and synonymously with terms such as attitude, disposition, opinion, perception, philosophy, and value. Because these various concepts are not directly observable and have to be inferred, and because of their overlapping nature, it is not easy to produce a precise definition of beliefs. (Leder & Forgasz, 2002, p. 96).

Different researchers associate belief with motivation and conception. Kloosterman (2002) sees the direct connection between belief and effort. ‘Student’s belief is something the student knows or feels that affects effort – in this case effort to learn mathematics’ (p. 248). Moreover, Kloosterman (2002) argues that student’s choices are on one hand based on beliefs and on the other hand on personal goals. Thus, there is a close connection between beliefs and choices. But sometimes the personal goals and the beliefs are at variance. One major example is the learning of mathematics. Many students believe that mathematics is boring, and strong effort is needed to learn it, but still find it important for life. This is a paradox. The reason for seeing mathematics as important can be practical – needs for a better profession and to some degree for a better life. ‘Most youngsters know, as an empirical and sociological fact, that mathematical competence – even if for unclear reasons – is a key to attractive education and job opportunities’ (Niss, 1994, p. 377). Jens Højgaard Jensen has marvellously expressed this idea in one sentence ‘Mathematics is useless to me, but at the same time I know that I am useless without mathematics’ (Niss, 1994, p. 377).

There have been two different notions about beliefs and conceptions in literature. In one case the beliefs are understood as a subclass of conceptions (Hart, 1989; Thompson, 1992) and on the other hand the conceptions are a subset of beliefs (Pehkonen, 1994). One can explain the concept of “conceptions” as an originations, comprehensions, ideas, rules, images etc. The easiest to understand and thus the widest, is the definition given by Rokeach (1972). He says: ‘a belief is any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase ‘I believe that…’” (p. 113).

Rejecting the McLeod (1989) idea to watch the person’s affective domain as an aggregate of beliefs, attitudes and emotions, there can be found the ideas which claim that belief is only one part of attitude in different researches (Aiken, 1980; Rokeach, 1972). More or less they see three aspects within an attitude: a cognitive component (beliefs and knowledge), an affective component (emotions, motivation, feelings) and a performance/behavioural component (actions). Here, the emotions are one component of attitude, and beliefs with knowledge are seen as a cognitive component of attitude.

As long as there are different people there will be dissimilar views about belief, attitude, emotions, meanings, mental images, concepts and so on. The definition does not play a major part in research, and thus every scientist will ascribe the importance of different aspects related to particular investigations. It means that the definition is affected by the questions and the motive of the research. Hence one cannot say that some definition is wrong and the other is right, they can be considered to be more or less suitable.
Relationship between beliefs and knowledge

The two parts of the individual – the affective domain and the cognitive domain - are inseparable and in complex connection. 'The main difficulty has been the inability to distinguish beliefs from knowledge, and the question is still unclarified' (Pehkonen, 1994, p. 27).

Thompson (1992) points out two ways to distinguish knowledge from belief – ‘degree of conviction’ (p. 129) and consensus. Firstly, beliefs can be held weakly or strongly. One can claim: “the new mathematics teacher is nice but she has not assessed us still so it can be changed” or “I know that the test in mathematics will be hard”. Beliefs can compartmentalize as something uncertain or certain, important or not so important. But one cannot say that one knows the fact weakly or strongly. Water starts boiling at a hundred degrees Celsius at sea level, and that’s it! Secondly, it is possible to believe something despite the awareness that the others do not agree with it and think about it differently. For example, “I believe gold can be found in North Pole”.

Underhill (1991) uses the word ‘knowing’ and takes the position that ‘knowing is believing’ (p. 20). Whatever one knows or does not know is simply the same that one believes and does not believe.

A. whenever we say we know something, we are simply asserting that we believe something, whether it is about quality called “red” or the being called “God” or the relationship “3+4=7”;

B. whatever we do not know, we may not know passively (we have no belief; we have never been exposed to it!), or negatively (we believe it’s opposite or some anti-belief or substitute belief; we believe something other than that);

C. all that we know reflects our beliefs based on empirical data or reason or faith; these might be thought to exist continuum.

(ibid, p. 21)

Saying that “the delta parameter of the option price shows how quickly the option price changes when the asset price is changing” only means that there is general belief in that. It can be because some trustable person (expert) has said so or because it has worked very well before, and why should it not work now (importance of the long experiences), or one does not believe that there is something else that will work better. Most scientific discoveries have started from the point that somebody believes in its validity and universality. If one does not believe in what one wants to prove then it is impossible to do that. We think the belief in rightness of doing research is one of the basic components of being scientist. To constructivists ‘knowledge without belief is contradictory’ (Confrey, 1990, p. 111).

One does not have to evaluate or justify the beliefs, it is something which belongs to the person. But there is definitely a need to explain the ideas related to knowledge because without justification it will not be accepted as knowledge. If one starts to justify beliefs then, according to Plato, the result will be the knowledge. ‘Knowledge is justified true belief’ (Mc Dowell, 1987, p. 94, and, in Furinghetti & Pehkonen, 2002, p. 42).
Differences between knowledge and beliefs can and cannot be seen. It all depends on how we define these two words. All knowledge is devised from human persons who believed in it, thus the claim: ‘All knowledge is a set of beliefs’ (Underhill, 1991, p. 5, emphasis original) can be professed as true.

**Belief systems**

There are different types of beliefs detached into different sections of questionnaires. This assortment is based on the idea of belief system. Green (1971) points out that individual’s beliefs are quasi-logically connected where the logic between the beliefs is defined individually. Talking about mathematics, then individual mathematics-related belief system is called his/her view of mathematics (Pehkonen, 1999; Schoenfeld, 1985). The human person’s belief system is dynamic, changeable and when individuals evaluate and assess their experiences and beliefs, then they are restructuring their system continuously (Thompson, 1992). Based on observations, Green (1971) has pointed out three dimensions of belief system where he emphasises the relations between the beliefs in system.

Firstly, the structure of the belief system is quasi-logical. Some beliefs are primary and some derivative. For example, a student believes that mathematics is useful for his/her life (this is a primary belief). And herewith s/he thinks that it is important (1) to work hard in mathematics class during the lessons, and (2) to work with problem-solving, and to try to relate the exercises to everyday life (these are the derivative beliefs).

Secondly, Green (1971) talks about central and peripheral beliefs in the system. Central beliefs are more important and held most strongly, whereas the peripheral ones can be changed more easily. In our view it is with experiences, practice and affirmation one’s own beliefs become more central. For instance, the newcomer teacher in school holds more peripheral beliefs, which are more changeable and negotiable. Experienced teachers’ beliefs are more central and thus more deep-rooted. Researchers have probably heard the sentence - “I have been teaching for 20 years, and I know precisely which method is effective” - several times during their studies. In some cases beliefs can be an insuperable obstacle for development. ‘Teachers’ conceptions of “good mathematics teaching” have been so deeply rooted that surface changes (changes in peripheral beliefs) – as changing curriculum, teaching materials, classroom management – can not influence them’ (Pehkonen, 1999, p. 7). The idea is not to underestimate the importance of experience, but emphasise the importance of flexibility and open-mindedness of others’ techniques and suggestions.

Thirdly, Green (1971) uses the word “cluster”. It means that beliefs are occurring in clusters, beliefs are not independent from each other. The clusters are in weak relationship or are not connected at all (Pehkonen, 1999). This phenomenon can explain the contradictions in pupils’ and teachers’ beliefs. Talking about mathematics related belief system there is a possibility to classify beliefs related to mathematics in several ways. Different researchers have different ways of grouping (Op’t Eynde, de Corte & Verschaffel, 2002) but broadly there are four sets of beliefs about mathematics
– beliefs about the nature of mathematics,
– beliefs about teaching and learning of mathematics,
– beliefs about the self in context of mathematics teaching and learning,
– beliefs about the nature of knowledge and the process of knowing.

The study

Methods and methodology

We plan to investigate students’ beliefs about mathematics teaching and learning. Our main project will take part in Estonia, presumably in spring 2006, but first we will carry out a pilot project in Norway. One of the main reasons is that there is a lack of investigations about beliefs in Estonia. After the studies we would like to make an analysis and compare the results between Estonia and Norway.

For now, we have done a small pilot study in Norway. We randomly chose one ordinary Norwegian class in grade 9 and gave the pupils the questionnaire. There were 25 pupils, who participated in this study. We used with permission the KIM-project questionnaire that was carried out in research in 1994, in order to be able to compare the results (Streitlien, Wiik & Brekke, 2001). Since our study was a pilot, the instrument was in main focus, and comparison is just showing a tendency besides a critical view on the function of the research instrument.

There were 112 questions divided into 13 groups, and these were beliefs about: mathematics as a subject; learning mathematics; own mathematical abilities; own experiences (security) during mathematics lesson; teaching of mathematics; learning a new topic in mathematics; environment in class; environment in school; differences between boys and girls; teaching tools in mathematics lesson; own evaluation for importance of mathematics; evaluation for teaching mathematics; mathematics and the future.

The questionnaire was based on the Likert–scales method. Likert- scales method is

a series of statements about the attitude object comprise a Likert scale. Items are typically rated from "strongly agree" to "strongly disagree" and five divisions are very common. Ease of administration and scoring of instruments using such items have ensured that this is a commonly used method for tapping attitudes and beliefs (Leder & Forgaz, 2002, p. 98).

The good points of using this method are easiness to make a conclusions (for example, calculate the average), the answers are concrete and the researcher does not have to struggle with answers (in written sentences), which are often hard to read and even more hard too understand and interprete. Thirdly, we call this method a “language free" method. It means that the answers, which the researcher will get are not based on knowledge of language, to carry out the investigation the researcher does not have to speak or understand the language in which the questionnaire is printed. As long as s/he has the translation of the questions s/he is able to collect the necessary data. Of course there are some
problems too using this method. One is the lack of ‘ability to explain how such beliefs formed or how beliefs are likely to influence action’ (Kloosterman, 2002, p. 262). In other words, one can get the answer to questions “what” and “which” but not “why” or “how”.

**Comparison**

In this part some of the results from our study are compared with the results from KIM-project. The numbers are given in percentages. For explanation, there are three lines in one row – first is the original line from questionnaire (in Norwegian), then our results and after that the results from the KIM-investigation. We have chosen to expose some sections of the questionnaire that are highly important. For reasons of space we cannot expose the full result in this paper.

The columns refers to respectively: totally agree, partially agree, uncertain, partially disagree, and totally disagree.

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Table 1

There is a small tendency of more positive thinking about mathematics in our group. More pupils think that mathematics is interesting and exciting. Pupils’ views are not so radical, they tend to be more uncertain and do not claim that mathematics is the subject which they like best/least. But nevertheless pupils agree to a high extent that mathematics is boring.
Children are often told that mathematics knowledge is useful, and sometimes necessary, especially when it comes to a future choice of profession. In KIM-study the students express a positive statement regarding the usefulness of mathematics. In both investigations almost all students agree (or were at least uncertain) that mathematics is useful in life. Now, more students affirm themselves: I will use mathematics after school. Fewer pupils are certain that they do not need to know mathematics.

### Table 2

Children are often told that mathematics knowledge is useful, and sometimes necessary, especially when it comes to a future choice of profession. In KIM-study the students express a positive statement regarding the usefulness of mathematics. In both investigations almost all students agree (or were at least uncertain) that mathematics is useful in life. Now, more students affirm themselves: I will use mathematics after school. Fewer pupils are certain that they do not need to know mathematics.
More pupils in our group think that mathematics suits them and is easy for them.

| Table 4 |
|------------------|---|---|---|---|
| 2h Å bli god I matematikk er avhengig av hardt arbeid. | 36 | 64 | 0 | 0 |
| To become good at mathematics is dependent on hard work. | 36 | 64 | 0 | 0 |
| To become good at mathematics is dependent on hard work KIM | 49 | 37 | 12 | 2 |
| 3f Jeg kan bli flink I matematikk hvis jeg lærer meg alle reglene. | 21 | 71 | 8 | 0 |
| I can become clever in mathematics if I learn all the rules. | 40 | 45 | 12 | 2 |
| I can become clever in mathematics if I learn all the rules KIM. | 21 | 71 | 8 | 0 |

**Diligence**

It is a high degree of agreement among the students that hard work is needed to become good at mathematics and that it is important to solve many tasks to remember the “procedures”. Both in our and in the KIM-study, there is a widespread conception among the students that diligence is crucial for learning mathematics. Even if the students stated that mathematics is quite easy, they are more aware that it needs much work, and they have to learn the rules to become good at mathematics.

**Conclusion**

The results from the KIM-study (Streitlien, Wiik & Brekke, 2001) are in principle confirmed by our small investigation. In our group there is a tendency to answer in a slightly more positive way. If this is just a coincidence or if it is a sign of a change over time in the attitudes and beliefs, must be studied in our further investigation. In conclusion we can bring out some interesting results:

- 96 % of students agree that mathematics is useful for life and 100 % agree that mathematics is important.
- 52 % claim that mathematics is boring, while 88 % are sure that they need to know mathematics.
- 84 % of pupils disagree that there is just one right answer in mathematics tasks.
- 96 % think that it is important to get good marks and 83 % find it important to cooperate in mathematics class. Everybody feels that it is important to know something about numbers and calculations and to know how to solve practical problems.
- Most pupils are good friends and have fun during classes and they are quite satisfied with the environment in class and school.

We can claim that most of the beliefs are going in positive direction from the KIM-study to our study. Students are aware the importance of mathematics in life. But there is still great emphasis on the rules and keeping in mind in mathematics, and 28 % of the students still think that it is innate to be good in mathematics.

As students claim that mathematics is important, it is unfortunate that school has not been able to arrange teaching is such a way that they also find it
challenging and fascinating. What is the reason that so many pupils find mathematics boring? How are the beliefs influencing pupils’ learning in mathematics? What can be changed in school to change this conception?

In our coming study we are going to relate pupils’ beliefs to their learning outcomes in order to find more knowledge about the relation between beliefs and learning. Pupils will answer both questionnaire about attitudes and beliefs and tests in mathematics, and some follow up interviews will be made. Some analysis of curricular content and working style will be considered in relation to beliefs. Our data will be compared to the earlier result from the KIM-study, and students in Norway will be compared to students in Estonia. Assessment and evaluation in mathematics must take place in full awareness of the importance of students’ attitudes and beliefs. The affective factors in the learning situation have important influence on the cognitive development of pupils.

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